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Electron–Phonon Interaction and Quantum Fluctuations in the Time-Dependent Damped Capacitance-Coupled Electric Circuit

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Starting from the electron–phonon interaction, the time-dependent capacitance-coupled electric circuit is quantized. Quantum fluctuations derived by this method are different from former ones.

KEY WORDS: electric circuit; phonon; quantum fluctuation.
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1. INTRODUCTION

With the rapid progress of nanophysics and nanoelectronics, the size of electric devices becomes smaller and smaller (Buot, 1993; Bulka and Stefanski, 2001; Hou *et al.*, 2001; Bobrov *et al.*, 2001; Garcia, 1992; Dobisz *et al.*, 1991). These developments may play an important role in providing the realization for future quantum computer (Lioyd, 1993; Makhlin *et al.*, 1999; Divincenzo *et al.*, 2000). When the scale of the electric materials reaches a characteristic dimension, say, the Fermi wavelength, quantum mechanical properties should be considered, since now the charge-carriers such as electrons exhibit quantum properties and the application of classical mechanics fails.

Many authors have already studied the quantum effects in the electric circuits (Louisell, 1973; Chen *et al.*, 1995; Zhang *et al.*, 2001; Choi, 2002; Song, 2003; Zhang and Liu, 2004). Louisell first quantized the LC (inductance-capacitance) circuit and discussed the quantum effects in it. Chen *et al.* (1995) quantized the equation of motion for the RLC (resistance-inductance-capacitance) circuit by introducing the complex charge and current. Zhang *et al.* (Zhang *et al.* 2001; Zhang and Liu, 2004) quantized the damped coupled circuits based on the classical equations of motion. In these treatments of the

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damped circuits, the mechanism of the generation of the resistance is not considered.

As is known, resistance is generated due to the collisions of the charge-carriers with the lattice oscillations. The lattice oscillations can be described by phonons. And each phonon may be simulated by a harmonic oscillator. Many phonons form an oscillator reservoir and the charge-carriers or electrons move in it. Such a picture of the circuit is similar to that of the damped oscillator (Yu and Sun, 1994; Caldeira and Leggett, 1983a,b), where the damped oscillator is considered as a harmonic oscillator in a heat reservoir of oscillators. Liang *et al.* (2002) showed that quantization of the damped oscillator by Yu and Sun (1994) can be applied to the time-dependent case

In this article, we study the damped time-dependent capacitance-coupled circuits with the mechanism of electron–phonon interaction included.

2. QUANTIZATION OF THE SYSTEM

The classical equations of motion for the capacitance-coupled circuit with time-independent parameters were given by Zhang *et al.* (2001). When the inductances and capacitances depend on time, the equations of motion is easy to get

$$\frac{d}{dt}\left(L_1\frac{dq_1}{dt}\right)+\frac{1}{C_1}q_1+\frac{1}{C}(q_1-q_2)+R_1\frac{dq_1}{dt}=\varepsilon_1 \tag{2.1}$$

$$\frac{d}{dt}\left(L_2\frac{dq_2}{dt}\right)+\frac{1}{C_2}q_2-\frac{1}{C}(q_1-q_2)+R_2\frac{dq_2}{dt}=\varepsilon_2 \tag{2.2}$$

where $q_i, L_i, C_i, R_i, \varepsilon_i, i=1,2$are the electric charges, inductances, capacitan ces, resistances and sources respectively, C is the capacitance in the coupling part. Using a_j^+, a_j to express the creation and annihilation operators for the jth mode phonon, two Hermitian operators can be defined

$$x_j=\sqrt{\frac{\hbar}{2\omega_j}}\frac{1}{i}(a_j^+-a_j) \tag{2.3}$$

$$p_j=\sqrt{\frac{\hbar\omega_j}{2}}(a_j^++a_j) \tag{2.4}$$

Clearly, x_j, p_j are the coordinate and momentum operators of a harmonic oscillator with unit mass and the commutation relation$[x_i, p_j]=i\hbar\delta_{ij}$ holds.

To recover the classical equations (2.1, 2.2) in the classical limit, we construct the Hamiltonian

$$H=\frac{p_1^2}{2L_1}+\frac{p_2^2}{2L_2}+\frac{q_1^2}{2C_1}+\frac{q_2^2}{2C_2}+\frac{(q_1-q_2)^2}{2C}-q_1\varepsilon_1-q_2\varepsilon_2$$

$$+\frac{1}{2}L_1(\Delta\omega_1)^2 q_1^2 + \frac{1}{2}L_2(\Delta\omega_2)^2 q_2^2$$
$$+q_1\sum_j C_j x_j + \sum_j\left(\frac{p_j^2}{2} + \frac{1}{2}\omega_j^2 x_j^2\right) + q_2\sum_l D_l y_l + \sum_l\left(\frac{p_l^2}{2} + \frac{1}{2}\omega_l^2 y_l^2\right) \tag{2.5}$$

where $p_i = L_i\dot{q}_i, i = 1, 2$ are the momenta, which are physically the magnetic fluxes in the inductances; $L_i(\Delta\omega_i)^2, i = 1,$ 2are the normalization constants (to be given in the following); x_j and y_l are the coordinates of the oscillators in the heat bath; C_j and D_l are the coupling constants between the charges (electrons) and oscillators (phonons). From (2.5), the Heisenberg equations of motion are derived

$$\frac{d}{dt}\left(L_1\frac{dq_1}{dt}\right) + \frac{1}{C_1}q_1 + \frac{q_1 - q_2}{C} + L_1(\Delta\omega_1)^2 q_1 + \sum_j C_j x_j = \varepsilon_1 \tag{2.6}$$

$$\frac{d}{dt}\left(L_2\frac{dq_2}{dt}\right) + \frac{1}{C_2}q_2 - \frac{q_1 - q_2}{C} + L_2(\Delta\omega_2)^2 q_2 + \sum_l D_l y_l = \varepsilon_2 \tag{2.7}$$

$$\ddot{x}_j = -\omega_j^2 x_j - C_j q_1 \tag{2.8}$$

$$\ddot{y}_l = -\omega_l^2 y_l - D_l q_2 \tag{2.9}$$

Solutions of (2.8, 2.9) can be found in Liang *et al.* (2002) or Yu and Sun (1994)

$$x_j(t) = x_{j0}\cos\omega_j t + \frac{\dot{x}_{j0}}{\omega_j}\sin\omega_j t - q_1(t)\frac{C_j}{\omega_j^2} + \frac{C_j}{\omega_j^2}\int_0^\infty \frac{\overline{q_1}s^2e^{st}}{2\pi i\left(s^2+\omega_j^2\right)}ds \tag{2.10}$$

$$y_l(t) = y_{l0}\cos\omega_l t + \frac{\dot{y}_{l0}}{\omega_l}\sin\omega_l t - q_2(t)\frac{D_l}{\omega_l^2} + \frac{D_l}{\omega_l^2}\int_0^\infty \frac{\overline{q_2}s^2e^{st}}{2\pi i\left(s^2+\omega_l^2\right)}ds \tag{2.11}$$

The footnote zero means initial value. Substituting (2.10) into (2.6), (2.11) into (2.7), we have

$$\frac{d}{dt}\left(L_1\frac{dq_1}{dt}\right) + \frac{1}{C_1}q_1 + \frac{q_1 - q_2}{C} + R_1\dot{q}_1 = f_1(t) + \varepsilon_1 \tag{2.12}$$

$$\frac{d}{dt}\left(L_2\frac{dq_2}{dt}\right) + \frac{1}{C_2}q_2 - \frac{q_1 - q_2}{C} + R_2\dot{q}_2 = f_2(t) + \varepsilon_2 \tag{2.13}$$

where

$$f_1(t) = -\sum_j C_j\left(x_{j0}\cos\omega_j t + \dot{x}_{j0}\frac{\sin\omega_j t}{\omega_j}\right) \tag{2.14}$$

$$f_2(t) = -\sum_l D_l \left(y_{l0} \cos \omega_l t + \dot{y}_{l0} \frac{\sin \omega_l t}{\omega_l} \right) \tag{2.15}$$

are the driving forces generated by the Brownian motion of oscillators in the heat bath. To get (2.12) and (2.13), the following spectrum densities are needed

$$\rho_1(\omega_j) = \frac{2R_1}{\pi} \frac{\omega_j^2}{C_j^2}, \quad \rho_2(\omega_j) = \frac{2R_2}{\pi} \frac{\omega_l^2}{D_l^2} \tag{2.16}$$

which were obtained by Caldeira and Leggett (1983a,b). The normalization constants are

$$L_1(\Delta\omega_1)^2 = \sum_j \frac{C_j^2}{\omega_j^2}, \quad L_2(\Delta\omega_2)^2 = \sum_l \frac{D_l^2}{\omega_l^2} \tag{2.17}$$

In the next section, we study the quantum fluctuations using (2.12) and (2.13).

3. QUANTUM FLUCTUATIONS AT FINITE TEMPERATURE

If there is no resistance, the equations of motion can be decoupled by the following transformations (Zhang *et al.* 2001, Setting the resistances there zero)

$$\begin{aligned} Q_1 &= \rho q_1 \cos\varphi - q_2 \tfrac{\sin\varphi}{\rho} \\ Q_2 &= \rho q_1 \sin\varphi + q_2 \tfrac{\cos\varphi}{\rho} \end{aligned} \tag{3.1}$$

where the angle φ and the parameter ρ satisfy

$$\tan 2\varphi = \frac{2\sqrt{L_1 L_2}}{L_2(1 + C/C_1) - L_1(1 + C/C_2)}, \quad \rho = \sqrt[4]{\frac{L_1}{L_2}} \tag{3.2}$$

If there is resistance, the condition that $R_1/L_1 = R_2/L_2 = \eta$ is assumed (Zhang *et al.*, 2001). For mathematical simplicity, we further assume that the inductances and the capacitances are proportional to each other respectively

$$L_1 \propto L_2, C_1 \propto C_2 \propto C \tag{3.3}$$

so that the angle φ and the parameter ρ are independent of time. Using the new coordinates in (3.1), Equations (2.12) and (2.13) are cast into the form

$$\begin{aligned} \frac{d}{dt}\left(M_2 \frac{dQ_2}{dt}\right) + M_2\eta \frac{dQ_2}{dt} + M_2\omega_2^2 Q_2 &= f_1(t) M_2 \frac{\rho \sin\varphi}{L_1} + f_2(t) M_2 \frac{\cos\varphi}{\rho L_2} \\ \frac{d}{dt}\left(M_1 \frac{dQ_1}{dt}\right) + M_1\eta \dot{Q}_1 + M_1\omega_1^2 Q_1 &= f_1(t) M_1 \frac{\rho \cos\varphi}{L_1} - f_2(t) M_1 \frac{\sin\varphi}{\rho L_2} \end{aligned} \tag{3.4}$$

where the "masses" and frequencies are

$$M_1 = M_2 = \sqrt{L_1 L_2} \tag{3.5}$$

$$M_1\omega_1^2 = \frac{1}{\rho^2}\left(\frac{1}{C}+\frac{1}{C_1}\right)\cos^2\varphi + \rho^2\left(\frac{1}{C}+\frac{1}{C_2}\right)\sin^2\varphi + \frac{\sin 2\varphi}{C} \quad (3.6)$$

$$M_2\omega_2^2 = \frac{1}{\rho^2}\left(\frac{1}{C}+\frac{1}{C_1}\right)\sin^2\varphi + \rho^2\left(\frac{1}{C}+\frac{1}{C_2}\right)\cos^2\varphi - \frac{\sin 2\varphi}{C} \quad (3.7)$$

Solution to each equation in (3.4) can be divided into two parts: The general solution of the homogeneous equation when the driving forces $f_1(t) = f_2(t) = 0$ and the special solution generated by the driving forces $f_1(t)$ and $f_2(t)$. The general solution of the homogeneous equation can be written as

$$Q_i^0(t) = A_{0i}\, g_i(t)\cos\left[\int_0^t \frac{d\tau}{m(\tau)g_i^2(\tau)} + \varphi_{0i}\right], \quad i = 1, 2 \quad (3.8)$$

where $m(t) = M(t)\exp(\eta t)$ and $g_i(t)$ satisfies

$$\ddot{g}_i + \frac{\dot{m}\dot{g}_i}{m} + \omega_i^2 g_i = \frac{1}{m^2 g_i^3} \quad (3.9)$$

Each of the homogeneous equations in (3.4) describes a time-dependent harmonic oscillator with mass $m(t)$ and frequency ω_i, momentum $P_i = m\dot{Q}_i$. The invariant of this system can be written as (Ji and Kim, 1996)

$$I_i = \hbar\omega_{0i}(b_i^+ b_i + 1/2) \quad (3.10)$$

where ω_{0i} is the frequency at the initial asymptotic region (Ji and Kim, 1996). The creation and annihilation operators are

$$b_i^+(t) = \frac{1}{\sqrt{2\hbar g_i^2}} Q_i(t) - i\frac{g_i}{\sqrt{2\hbar}}\left[P_i(t) - \frac{m\dot{g}_i}{g_i}Q_i(t)\right]$$

$$b_i(t) = \frac{1}{\sqrt{2\hbar g_i^2}} Q_i(t) + i\frac{g_i}{\sqrt{2\hbar}}\left[P_i(t) - \frac{m\dot{g}_i}{g_i}Q_i(t)\right] \quad (3.11)$$

We write the eigenstate of the invariant (3.10) as $|n_i\rangle$, $n_i = 0,\ 1,\ 2, \ldots$.

To consider the temperature effect, we extend the thermo field dynamics (TFD) (Umezawa and Yamanaka, 1988; Kireev *et al.*, 1989) to the time-dependent case. According to Ji and Kim (1996), using the invariant theory the formulas of quantum statistics are similar to that of time-independent systems. Assuming that initially a system is at thermal equilibrium, at finite temperature the un-normalized density operator is $\rho_i = \exp[-I_i/(kT)]$ with k being the Boltzmann constant and T the temperature. The mean particle number is

$$n_{0i} = Tr(b_i^+ b_i \rho_i)/Tr\rho_i = \left(\exp\frac{\hbar\omega_{0i}}{kT} - 1\right)^{-1} \quad (3.12)$$

In TFD, the operators b_i^+, b_i, acting on the actual Hilbert space, are associated with the thermal freedom operators $\tilde{b}_i^+$, $\tilde{b}_i$ in the extended Hilbert space, a fictitious space or tilde space. The operators $\tilde{b}_i^+$, $\tilde{b}_i$ commute with b_i^+, b_i and satisfy the commutation relation $[\tilde{b}_i, \tilde{b}_i^+] = 1$. The whole space of the system is the direct product of the actual Hilbert space and the tilde space. Any state in TFD will be two-mode, for instance the two-mode Fock state $|n_i\tilde{n}_i\rangle = |n_i\rangle\,|\tilde{n}_i\rangle$, etc. The theory of TFD demands $n_i = \tilde{n}_i$ in number. The state at finite temperature is related to the state at zero temperature through the operator

$$T(\theta) = \exp[-\theta(b_i\tilde{b}_i - b_i^+\tilde{b}_i^+)] \tag{3.13}$$

where $\sinh^2\theta = n_{0i}$. The two-mode Fock state at finite temperature is $T(\theta)\,|n_i\tilde{n}_i\rangle$. For a physical quantity Ω, the average $\langle n_i|\,\Omega\,|n_i\rangle$ becomes $\langle n_i\tilde{n}_i|\,T^+(\theta)$ $\Omega T(\theta)\,|n_i\tilde{n}_i\rangle$. Using (3.13), it is not difficult to show that

$$\begin{aligned} T^+(\theta)b_iT(\theta) &= ub_i + v\tilde{b}_i^+, \\ T^+(\theta)b_i^+T(\theta) &= ub_i^+ + v\tilde{b}_i. \end{aligned} \tag{3.14}$$

where

$$\begin{aligned} u &= \sqrt{1+n_{0i}} = \cosh\theta, \\ v &= \sqrt{n_{0i}} = \sinh\theta. \end{aligned} \tag{3.15}$$

The quantum fluctuations for the Fock state are easily obtained

$$\begin{aligned} \left\langle(\Delta Q_i^0)^2\right\rangle &= \left(n_i + \tfrac{1}{2}\right)\hbar g_i^2 \coth\frac{\hbar\omega_{0i}}{2kT}, \\ \left\langle(\Delta P_i^0)^2\right\rangle &= \left(n_i + \tfrac{1}{2}\right)\hbar\left[\frac{1}{g_i^2} + (m\dot{g}_i)^2\right]\coth\frac{\hbar\omega_{0i}}{2kT}. \end{aligned} \tag{3.16}$$

where $P_i^0 = m\dot{Q}_i^0$. If the inductances and capacitances are independent of time, so will be M and ω_i. In this case, (3.9) has the solution $g_i(t) = \left[\left(\omega_i^2 - \eta^2/4\right)M^2\right]^{-1/4}$ $\exp(-\eta t/2)$and the quantum fluctuations of the charges Q_i^0 and currents $\dot{Q}_i^0 = P_i^0/m$ tend to zero in the long time limit. It is also easy to see that, in the long time limit, the solution (3.8) and its time derivative both go to zero. Now the solution and quantum fluctuations of the charges and currents are determined by the special solution induced by the driving forces $f_1(t)$ and $f_2(t)$

$$\begin{aligned} Q_1 &= Q_x(t)\frac{\rho\sin\varphi}{L_1} + Q_y(t)\frac{\cos\varphi}{\rho L_2} \\ Q_2 &= q_x(t)\frac{\rho\cos\varphi}{L_1} - q_y(t)\frac{\sin\varphi}{\rho L_2} \end{aligned} \tag{3.17}$$

where $q_x(t)$, $Q_x(t)$ and $q_y(t)$, $Q_y(t)$ are generated by $f_1(t)$and$f_2(t)$respectively, which have the following forms

$$Q_x(t) = \sum_j (b_{j1}x_{j0} + b_{j2}\dot{x}_{j0}) \tag{3.18a}$$

$$Q_y(t) = \sum_j (b_{j1}y_{j0} + b_{j2}\dot{y}_{j0}) \tag{3.18b}$$

$$q_x(t) = \sum_j (d_{j1}x_{j0} + d_{j2}\dot{x}_{j0}) \tag{3.18c}$$

$$q_y(t) = \sum_j (d_{j1}y_{j0} + d_{j2}\dot{y}_{j0}) \tag{3.18d}$$

The coefficients are

$$\begin{aligned} b_{j1} &= -\frac{C_j}{\left(\omega_1^2 - \omega_j^2\right)^2 + \eta_1^2\omega_j^2}\left[\left(\omega_1^2 - \omega_j^2\right)\cos\omega_j t + \eta\omega_j \sin\omega_j t\right] \\ b_{j2} &= -\frac{C_j}{\left(\omega_1^2 - \omega_j^2\right)^2 + \eta_1^2\omega_j^2}\left[-\eta\cos\omega_j t + \frac{\omega_1^2 - \omega_j^2}{\omega_j}\sin\omega_j t\right] \end{aligned} \tag{3.19a}$$

$$\begin{aligned} d_{j1} &= -\frac{C_j}{\left(\omega_2^2 - \omega_j^2\right)^2 + \eta^2\omega_j^2}\left[\left(\omega_2^2 - \omega_j^2\right)\cos\omega_j t + \eta\omega_j \sin\omega_j t\right] \\ d_{j2} &= -\frac{C_j}{\left(\omega_2^2 - \omega_j^2\right)^2 + \eta^2\omega_j^2}\left[-\eta\cos\omega_j t + \frac{\omega_2^2 - \omega_j^2}{\omega_j}\sin\omega_j t\right] \end{aligned} \tag{3.19b}$$

Carrying out the time derivative, we get the currents from (3.17–3.19). Thus, quantum fluctuations of the charges and currents can be computed. To save space, we focus our attention on the quantum fluctuations of the charges. Use the method by Yu and Sun (1994) or Liang *et al*. (2002), we obtain

$$\begin{gathered} \left\langle \Delta Q_x^2 \right\rangle = L_1 \left\langle \Delta Q_0^2 \right\rangle, \left\langle \Delta q_x^2 \right\rangle = L_1 \left\langle \Delta q_0^2 \right\rangle \\ \left\langle \Delta Q_y^2 \right\rangle = L_2 \left\langle \Delta Q_0^2 \right\rangle, \left\langle \Delta q_y^2 \right\rangle = L_2 \left\langle \Delta q_0^2 \right\rangle \\ \langle (q_y Q_y + Q_y q_y) \rangle = (L_2/L_1)\, \langle (q_x Q_x + Q_x q_x) \rangle \end{gathered} \tag{3.20}$$

where

$$\left\langle \Delta Q_0^2 \right\rangle = \frac{\hbar}{2\pi\sqrt{\omega_1^2 - \eta^2/4}}\left(\frac{\pi}{2} + \alpha_1\right), \alpha_1 = \arctan\frac{\left(\omega_1^2 - \eta^2/2\right)}{\eta\sqrt{\omega_1^2 - \eta^2/4}} \tag{3.21a}$$

$$\left\langle \Delta q_0^2 \right\rangle = \frac{\hbar}{2\pi\sqrt{\omega_2^2 - \eta^2/4}}\left(\frac{\pi}{2} + \alpha_2\right), \alpha_2 = \arctan\frac{\left(\omega_2^2 - \eta^2/2\right)}{\eta\sqrt{\omega_2^2 - \eta^2/4}} \tag{3.21b}$$

The inverse transformations of (3.1) are

$$q_1 = \frac{1}{\rho}(Q_2 \sin\varphi + Q_1 \cos\varphi)$$
$$q_2 = \rho(Q_2 \cos\varphi - Q_1 \sin\varphi) \tag{3.22}$$

After some calculations, quantum fluctuations of the charges are derived

$$\langle \Delta q_1^2 \rangle = \frac{1}{L_1}\left[\langle \Delta q_0^2 \rangle \sin^2\varphi + \langle \Delta Q_0^2 \rangle \cos^2\varphi\right]$$
$$\langle \Delta q_2^2 \rangle = \frac{1}{L_2}\left[\langle \Delta q_0^2 \rangle \cos^2\varphi + \langle \Delta Q_0^2 \rangle \sin^2\varphi\right] \tag{3.23}$$

In the limit $\eta \to 0$, the parameters $\alpha_1 \to \pi/2$, $\alpha_2 \to \pi/2$, and so $\langle \Delta Q_0^2 \rangle = \frac{\hbar}{2\omega_1}$, $\langle \Delta q_0^2 \rangle = \frac{\hbar}{2\omega_2}$. Under these conditions, the quantum fluctuations reduce to

$$\langle \Delta q_1^2 \rangle = \frac{\hbar}{2L_1}\left[\frac{\sin^2\varphi}{\omega_2} + \frac{\cos^2\varphi}{\omega_1}\right]$$
$$\langle \Delta q_2^2 \rangle = \frac{\hbar}{2L_2}\left[\frac{\cos^2\varphi}{\omega_2} + \frac{\sin^2\varphi}{\omega_1}\right] \tag{3.24}$$

which are the quantum fluctuations of the undamped capacitance-coupled circuit at the ground state (See (4.1) and (4.2) in Zhang *et al.* (2001) when the damping factor is zero).

4. CONCLUSIONS

As a conclusion, we compare our results with former ones. In the work of Zhang *et al.* (2001), quantum fluctuations of both the charges and currents are zero in the long time limit (See their equations (4.1) and (4.2)). The quantum fluctuations of the currents are zero can be seen this way. From their Hamiltonian (2.3), one gets the currents $j_i = dq_i/dt = (p_i/L_i)\exp(-\eta t)$. The quantum fluctuation of the current can be expressed by the quantum fluctuation of the generalized momentum $\langle \Delta j_i^2 \rangle = (\langle \Delta p_i^2 \rangle / L_i^2)\exp(-2\eta t)$. From their equations (4.3) and (4.4), one can see that quantum fluctuations of the currents really tend to zero after a long time. These conclusions agree with the part derived from the homogeneous form of (3.4) in our calculation. The results of Chen *et al.* (1995) correspond to $\alpha_1 = \alpha_2 = \pi/2$. As α_1 and α_2 are usually less than $\pi/2$, the quantum fluctuations obtained here are smaller than that derived by the way of Chen *et al.* (1995).

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